



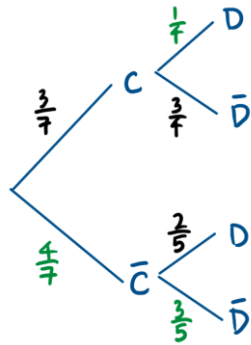
Mathematics Advanced Student Workbook – Solutions

inspired.edu.au

Statistical Analysis Probability

- Q1. (a) $a = 0.34, b = 0.18$
 (b) $\text{Var}(X) = 2.0075$

- Q2. (i)



- (ii) (a) $\frac{3}{28}$
 (b) $\frac{23}{35}$

- Q3. (i)

$$\sum_{i=0}^4 p_i = 1$$

$$0.2 + 0.6p^2 + 0.1 + (1 - p) + 0.1 = 1$$

$$0.6p^2 - p + 0.4 = 0$$

$$6p^2 - 10p + 4 = 0$$

$$(6p - 4)(p - 1) = 0$$

$$p = \frac{2}{3} \text{ or } p = 1$$

(ii) $E(X) = \frac{28}{15}$

(iii) $P(X \geq E(X)) = \frac{8}{15}$

Exponential Function

Q1. $-\frac{1}{3}e^{1-3x} + C$

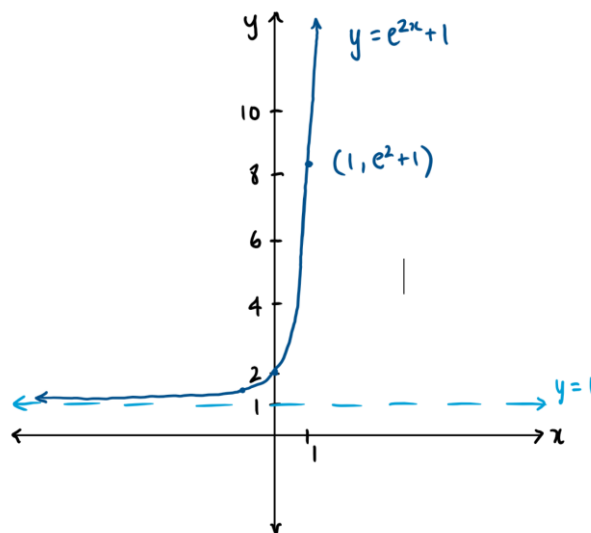
Q2. $\frac{1}{8}e^{8x} + \frac{1}{2}e^{4x} + x + C$

Q3. $\frac{4xe^{2x}}{(2x+1)^2}$

Q4. $-\frac{1}{\pi} \sin\left(\frac{x}{\pi}\right) \cdot e^{\cos x}$

Q5. $x = 0$ or $x = \ln 4$

Q6. (i)



(ii) $\frac{3}{2} - \ln 2$

Q7. $\left(\frac{1}{3} \ln 2, 5\right)$

Logarithmic Function

Q1. $f(x) = 2 \ln x + 1$

Q2. (i)
$$\begin{aligned} 6 + \frac{6x}{x^2 + 1} \\ &= \frac{6(x^2 + 1)}{x^2 + 1} + \frac{6x}{x^2 + 1} \\ &= \frac{6x^2 + 6x + 6}{x^2 + 1} \end{aligned}$$

(ii) $6 + 3 \ln 2$

Q3. $\frac{5}{x}(\ln x + 4)^4$

Q4. $\frac{1}{2} \ln(4x^2 + 3) + C$

Q5. $y = 2x$

Q6. $A = 18 - \frac{8}{\ln 3}$

Exponential Equations

Q1. 152,684 people.

Q2. (i) $C \rightarrow 0$

(ii) $C'(t) = -10(e^{-t} - 2e^{-2t})$

(iii) $t = \ln 2$ minutes

Q3. (i) 34.07°C

(ii) Cooling at $2.17^\circ\text{C}/\text{min}$

(iii) 11 minutes and 37 seconds

Q4. (i) When $t = 0$, $A = 0.24$:

$$0.24 = A_0 \times e^0$$

$$\therefore A_0 = 0.24$$

(ii) Sunday 8 am

(iii) Decreasing at 0.008 mg/mL

Maxima/Minima Questions

Q1. (i) $A = 200$

$$2xy - \frac{1}{2}\pi \times x^2 = 200$$

$$2xy = 200 - \frac{\pi x^2}{2}$$

$$y = \frac{100}{x} - \frac{\pi x}{4}$$

(ii) $x = \frac{20}{\sqrt{4+3\pi}}$ (or 5.46 cm)

Q2. (i) Let t be the number of hours after $t = 0$.

$$AC = 40 - 20t \quad \text{for } 0 \leq t \leq 2$$

$$AB = 20t$$

$$d^2 = (40 - 20t)^2 + (12t)^2 - 2(40 - 20t)(12t) \cos 120^\circ$$

$$= 1600 - 1600t + 400t^2 + 144t^2 - (960t - 480t^2) \left(-\frac{1}{2}\right)$$

$$d^2 = 304t^2 - 1120t + 1600$$

(ii) 23.84 cm

Trigonometry – Calculus

- Q1. (i) $3 \cos x (\sin x + 1)$
(ii) $\cos(2x) - 2x \sin(2x)$
(iii) $10 \tan(5x + 1) (\tan^2(5x + 1) + 1)$
- Q2. $\frac{\sqrt{3} - 1}{4} - \frac{\pi}{48}$
- Q3. 4
- Q4. (i) Let $4 \sin(2x) = -4 \cos(2x)$ for $0 \leq x \leq \frac{\pi}{2}$
 $\tan(2x) = -1$ for $0 \leq 2x \leq \pi$
 $2x = \pi - \frac{\pi}{4}$
 $x = \frac{3\pi}{8}$
(ii) $2\sqrt{2} + 2$

Rates of Change – Motion

- Q1. (i) $\dot{x}(t) = 3t^2 - 9t - 12$
(ii) $t = 4$ seconds
(iii) $d = 542$ metres

- Q2. (i) $a = 8e^{-2t} + 3e^{-t}$

$$\begin{aligned}v &= \int 8e^{-2t} + 3e^{-t} dt \\&= -4e^{-2t} - 3e^{-t} + C\end{aligned}$$

$$\text{When } t = 0, \quad v = -6$$

$$-6 = -4e^0 - 3e^0 + C$$

$$C = 1$$

$$v = -4e^{-2t} - 3e^{-t} + 1$$

$$\begin{aligned}x &= \int -4e^{-2t} - 3e^{-t} + 1 dt \\&= 2e^{-2t} + 3e^{-t} + t + C\end{aligned}$$

$$\text{When } t = 0, \quad x = 5$$

$$5 = 2e^0 + 3e^0 + 0 + C$$

$$C = 0$$

$$\therefore x = 2e^{-2t} + 3e^{-t} + t + C$$

- (ii) $t = \ln 4$ seconds

- (iii) $x = \frac{7}{8} + \ln 4$ metres

- Q3. (i) $\dot{x} = 4 \cos(2t) - 2$
 $\ddot{x} = -8 \sin(2t)$

- (ii) $t = \frac{\pi}{6}, \frac{5\pi}{6}$ seconds

- (iii) $d = \frac{4\pi}{3} + 1$ metres

- (iv) 8 m/s^2

Investment, Annuities, and Loans

Q1. 5 years and 2 months

Q2. (i) \$20 616.93

(ii) \$7 029.69

Q3. (i) $r = \frac{0.18}{12} = 0.015$

$$A_1 = 30\,000(1 + 0.015) - 900$$

$$A_2 = A_1(1.015) - 900$$

$$\therefore A_2 = 30\,000(1.015)^2 - 900(1 + 1.015)$$

$$\begin{aligned} \text{(ii)} \quad A_3 &= A_2(1.015) - 900 \\ &= 30\,000(1.015)^3 - 900(1 + 1.015 + 1.015^2) \end{aligned}$$

$$\begin{aligned} A_n &= 30\,000(1.015)^n - 900(1 + 1.015 + 1.015^2 + \dots + 1.015^{n-1}) \\ &= 30\,000(1.015)^n - 900 \times \frac{1(1.015^n - 1)}{1.015 - 1} \\ &= 30\,000(1.015)^n - 60\,000(1.015^n - 1) \\ \therefore A_n &= 60\,000 - 30\,000(1.015)^n \end{aligned}$$

(iii) 47 months

(ii) $2\sqrt{2} + 2$



These materials are the copyright property of InspirationED.

No part of this publication can be reproduced, copied, scanned, stored in a retrieval system, communicated, transmitted or disseminated in any form or by any means without the prior written consent of InspirationED. Reproduction of the whole or part of this document constitutes an infringement in copyright and can result in legal action.

