Solutions to Dimensional Analysis Problems

1. The conversion factors needed here are:

$$1 \ km^{2} = 1 \times 10^{9} \ m^{3}$$
$$1 \ m^{3} = 1 \times 10^{6} \ cm^{3}$$
$$1 \ cm^{3} = 1 \times 10^{-3} \ L$$

Now, we would like to find out how many litres are in 1 cubic kilometre. We do this as follows:

$$1 \ km^{3} \times \frac{1 \times 10^{9} \ m^{3}}{1 \ km^{3}} \times \frac{1 \times 10^{6} \ cm^{3}}{1 \ m^{3}} \times \frac{1 \times 10^{-3} \ L}{1 \ cm^{3}}$$
$$= 1 \ km^{3} \times \frac{1 \times 10^{9} \ m^{3}}{1 \ km^{3}} \times \frac{1 \times 10^{6} \ cm^{3}}{1 \ m^{3}} \times \frac{1 \times 10^{-3} \ L}{1 \ m^{3}}$$

On simplifying this, we find that $1 \ km^3 = 1 \times 10^{12} \ L$. So,

$$1.35 \times 10^9 \ km^3 = 1.35 \times 10^9 \times 1 \times 10^{12} \ L$$
$$= 1.35 \times 10^{21} \ L$$

That is, the volume of water in the world's oceans is approximately 1.35×10^{21} litres.

2. The conversion factors are given:

1 barrel = 42 gallons
1 gallon = 4 quarts
1 quart =
$$9.46 \times 10^{-4} m^3$$

Now we would like to find out how many cubic metres are in 1 barrel. We do this as follows:

$$1 \text{ barrel} \times \frac{42 \text{ gallons}}{1 \text{ barrel}} \times \frac{4 \text{ quarts}}{1 \text{ gallon}} \times \frac{9.46 \times 10^{-4} m^3}{1 \text{ quart}}$$
$$= 1 \text{ barrel} \times \frac{42 \text{ gallons}}{1 \text{ barrel}} \times \frac{4 \text{ quarts}}{1 \text{ gallon}} \times \frac{9.46 \times 10^{-4} m^3}{1 \text{ quart}}$$

On simplifying this, we find that:

1 barrel =
$$3269.28 \times 10^{-4} m^3$$

= $3.26928 \times 10^3 \times 10^{-4} m^3$
= $3.26928 \times 10^{-1} m^3$

So,

$$9.6 \times 10^8 |textbarrels = 9.6 \times 10^8 \times 3.26928 \times 10^{-1} m^3$$

= 31.385088 × 10⁷ m³
= 3.1385088 × 10⁸ m³

That is, the amount in cubic metres is 3.139×10^8 .