A tale of two diagnostic tests

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A problem

> *It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, . . .*

Charles Dickens. "A tale of two cities."

For Australians wanting to study at university it has never been better—entry is easier than ever before.

For many of us teaching undergraduate students it's never been harder as we have to deal with many students who are very poorly prepared for university study. The diversity of backgrounds is enormous.

Students are choosing less mathematics at secondary school.

Many universities have no mathematics requirements for entry.

Inadequate mathematics preparation for university

Of domestic students enrolled in our first-year mathematics and statistics subjects in 2016, in the final 2 years of secondary school

- *50.5%* did no mathematics,
- a further *15%* studied the lowest level of mathematics.

So *65.5% of our first-year students are not prepared for their mathematics and statistics studies*.

We find that many of our students lack very basic mathematical skills.

Students need to have basic skills before moving on. Mathematics is cumulative.

Diagnostic mathematics tests

Conjecture

Diagnostic tests can be useful.

Such tests are becoming more common. (For us anyway.)
The focus is diagnostic, placement, readiness or competency tests.

- Often used when students arrive at university to gauge their basic skills in mathematics,
- Usually short,
- Usually cover very basic mathematics,
- Often run because of the weak mathematical skills of some or many students,
- Some are compulsory, some are not,
- Some are paper-based, some online,
- Some are for credit, some are not.

Why use a diagnostic test?

1. Inform teaching staff about the level of knowledge of students, perhaps enabling them to target their teaching to the level of (most) students.
2. Identify at-risk students, with the aim of providing assistance.
3. Inform students of any deficiencies in their knowledge so that they can then address these.
4. Enable students and/or staff to decide on the right level of subject for each student (in cases where there is a choice).
5. Require students to reach a determined level of skills in order to progress.
6. Predict performance.
7. Inform non-mathematicians and decision makers about the level of mathematical knowledge of students.

Design students

In 2016 approximately 70 first-year industrial design students enrolled in a basic mathematics subject.
The test:

- 10 minutes, 14 short answer questions, paper,
- students must get 11/14 in order to pass the subject,
- six attempts throughout the semester were allowed. A slightly different test was used for each attempt.
- The test contributed 10% to the final mark for the subject.

The purposes:

3. Inform students of any deficiencies in their knowledge so that they can then address these.
5. Require students to reach a determined level of skills in order to progress.
7. Inform non-mathematicians and decision makers about the level of mathematical knowledge of students.

The test (no calculators)

- Find $-6 + 4 \times -5 - 3$.
- Write $\frac{11}{20}$ as a percentage.
- Find 4% of \$6000.
- Find $\frac{2}{3} + \frac{3}{5}$.
- Find $6.32 \div 100$.
- Find $\frac{3}{8} \times 1\frac{1}{3}$.
- Round 27.48281 to 3 significant figures.
- Arrange the following in ascending order (smallest to largest)

  0.702, 0.072, 0.72, 0.0702.
- Change 24.59 metres into centimetres.
- Write 0.000483 using scientific notation.
- Simplify $5x - 3w - 7x + 8w$.
- Simplify $3x \times 2xy$.
- Expand and simplify $3(a + 4) + 4a$.
- A photograph has length 24 cm and width 16 cm. It is to be enlarged so that its length becomes 36 cm. What will be the width?

Engineering students

The test:

- 50 minutes, in university computer labs,
- 50 multiple choice questions,
- students must get 70% or more to get into Mathematics 1, else they must do a preliminary subject,
- The test is not for credit.

The purposes:

4. Enable staff to decide on the right level of subject for each student.

Sample questions

Test topics:

- exponents,
- factorisation,
- linear equations,
- surds,
- exponential and logarithmic equations,
- trigonometry,
- functions,
- graphs,
- differentiation, and integration.

Sample Questions:

Simplify $\left(2x^{-2}\right)^{-2}$.

- $\frac{x^4}{4}$
- $2x^4$
- $\frac{2}{x^4}$
- $4x^4$

Factorise completely $3x^2 + 6xy + 9x^3$.

- $x(3x + 6y + 9x^2)$
- $3(x^2 + 2xy + 3x^3)$
- $3x(x + 2y + 3x^2)$
- $3x^2(1 + 2y + 3x)$

Sample questions (cont)

If $f(x) = x^3 + 1$ and $g(x) = \sqrt{x}$, determine $g\left(f(2)\right)$.

- $\sqrt{2}$
- 3
- $\sqrt{3}$
- 1

Solve $\log_3 x = -2$.

- $x = -\frac{1}{6}$
- $x = \frac{1}{9}$
- $x = -\frac{1}{9}$
- no solution

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ given $y = 3x^{-4}$.

- $3x^{-5}$
- $-12x^{-5}$
- $-7x^{-3}$
- $-12x^{-3}$

Find $\displaystyle\int_1^3 \left(x^{-2} - 1\right)\, dx$.

- $-\frac{4}{3}$
- $-x^{-1} - x + c$
- $-\frac{4}{3} + c$
- $\frac{4}{3}$

Outcomes

Though the two tests were both short tests of basic mathematics and were both administered near the start of students' university studies, the tests had different purposes and thus the measures used to determine success or otherwise of these tests were different.

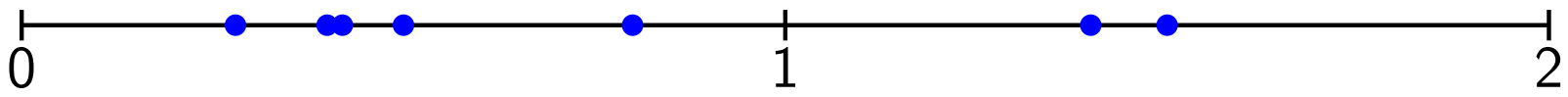
Outcomes, design students

The first attempt:

- 56% did not reach the threshold.
- Almost a quarter of students
  - could not change a simple measurement from m into cm,
  - could not divide an integer by 100;
- Over a quarter could not add 2 simple fractions;
- Over a third could not simplify $-6 + 4 \times -5 - 3$;

After a few attempts:
Some do not know where the decimal point goes in 17.

Mark on the number line where $\frac{2}{5}$ should be (5th test, 8 students).



There is a problem; this is evidence.

Outcomes, design students

- Of the 61 students who were still enrolled at the end of semester all attempted the test at least once.
- Attempt 1: 28 of 60 reached the threshold.
- 11 had not reached the threshold by the end of semester. That's 18%.
- 50 eventually reached the threshold. Of these, 7 failed (4 by missing the exam).
- A noticeable number of students spent time working on basic skills. Not all of these made it, but they did improve.
- *"I would also like to thank you for your effort this semester, I feel as if I have learned something (finally) although there is still a ways to go."* [unsolicited email]

Was the test useful? Yes!

- It did improve the basic skills of some.
- Those who didn't reach the threshold failed anyway.

Purposes 3, 5, 7 + 1, 6.

Outcomes, engineering students

Students who score $< 70\%$ in the test must pass a preliminary subject before attempting Mathematics 1.

- A reduction in the failure rate for Mathematics 1 and a reduction in the number of students who fail this subject multiple times.
- The failure rate for Mathematics 1 was previously regularly above 40% with occasional peaks at over 50%; it is now typically about 30%.
- The failure rate for Preliminary Mathematics remains at about 40%. Of this 40% about half failed at least one other subject in the semester.

Raise the threshold?
Was the test successful? Yes!
Purposes 4 + 1, 6.

There are many good reasons to use a diagnostic test

1. Inform teaching staff about the level of knowledge of students, perhaps enabling them to target their teaching to the level of (most) students.
2. Identify at-risk students, with the aim of providing assistance.
3. Inform students of any deficiencies in their knowledge so that they can then address these.
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