The mathematics problem
or
Falling standards of Australian mathematics

Leanne Rylands

Centre for Research in Mathematics and Data Science,
and
Mathematics Education Support Hub

Western Sydney University

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Outline

- The mathematics problem
- Evidence of a problem
- Consequences of the problem
- Digging down

Since giving this talk I've added a few bits and pieces as well as references at the end. This is so that you can easily use the evidence presented here should you want to inform others of the maths problem.

The mathematics problem

Students are coming to university under-prepared for the mathematical and statistical aspects of their studies.

In the UK
Reports of the mathematics problem go back as far as at least 1995.

1997: "Concerns have been expressed nationally at the level of mathematical expertise demonstrated by undergraduate engineering students from as long ago as 1978."

Crowther et al.

The mathematics problem

Students are coming to university under-prepared for the mathematical and statistical aspects of their studies.

2000: The UK has a mathematics problem.

Measuring the mathematics problem:

- There is a need for each mathematics department to understand the problem—run diagnostic tests for new undergraduates.
- There is a need for the mathematics community and its stakeholders to speak with one voice.

Measuring the mathematics problem

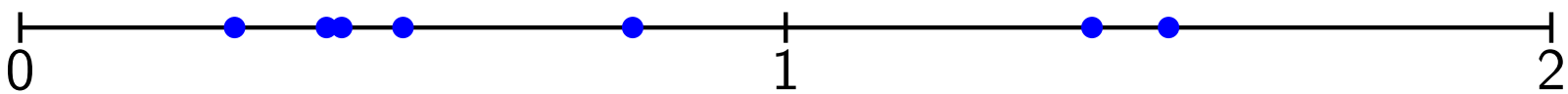
Do we have a problem?

A test in a first-year mathematics subject. Calculators were not allowed.

- Almost a quarter of students
  - could not change a simple measurement from m into cm,
  - could not find 4% of \$6000,
  - could not divide an integer by 100;
- Over a quarter could not add 2 simple fractions;
- Over a third could not simplify $-6 + 4 \times -5 - 3$.

On further examination

- A few students did not know where the decimal point is in 17
- Some could not place $\frac{2}{5}$ on the number line shown.

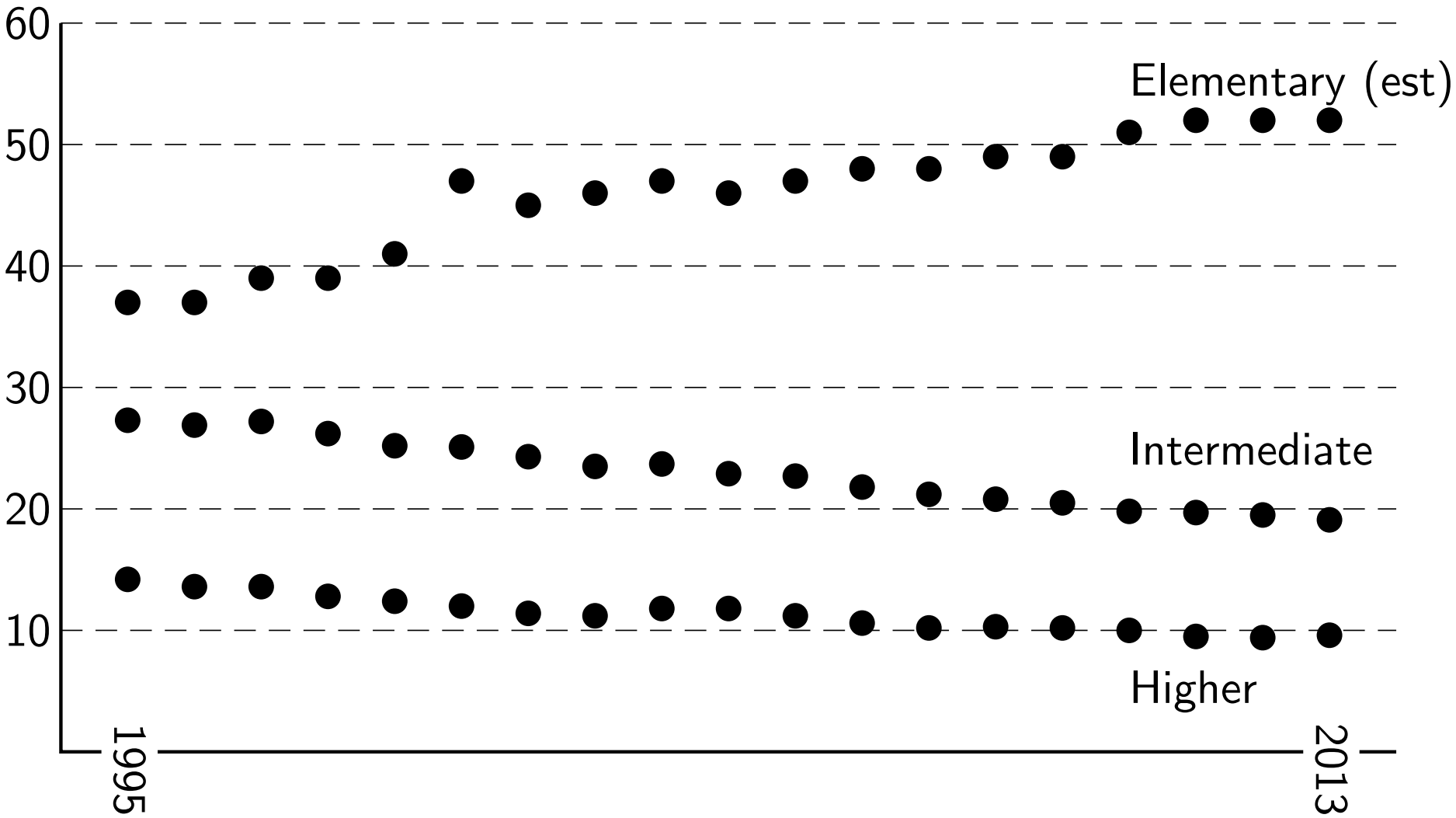


If there is a problem, then we need evidence.

Evidence for

- each of us,
- our community,
- universities,
- stakeholders,
- Australia.

Australia: less high school mathematics, 1995–2013



Barrington and Brown (2014)

Australia: less high school mathematics, 1990-1995

1990–1995

- Higher maths enrolments dropped, approx 23%→18%
- Intermediate maths enrolments had a small decrease
- Elementary maths enrolments increased, approx 30%→44%.

Forgasz (2006) Figs 1.8–1.10

Before 1990
Year 12 retention rates almost doubled from around 30% in 1980 to around 60% in 1989. (p.19)

Forgasz (2006)

NSW: less high school mathematics

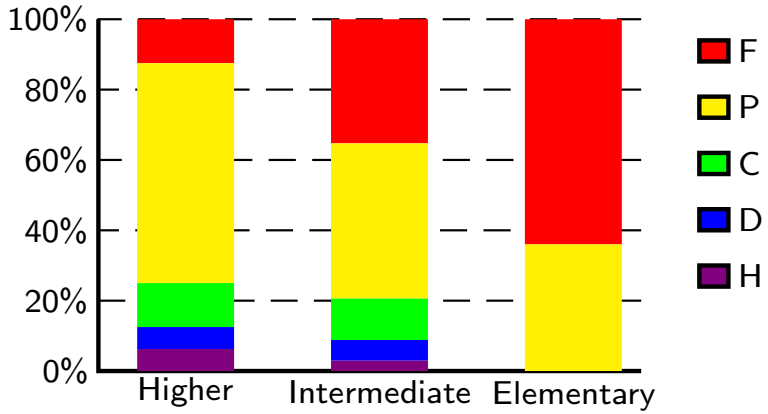
NSW Mathematics participation for the Higher School Certificate (HSC) by highest level for 1992, 2002 and 2012.

		Elementary	Intermediate	Higher	
	No maths	Gen. Maths	2 unit Maths	Ext. 1	Ext. 2
1992	1.8%	36.6%	36.2%	17.5%	7.7%
2002	6.8%	49.8%	25.9%	11.8%	5.7%
2012	15.8%	49.2%	18.9%	9.8%	6.3%

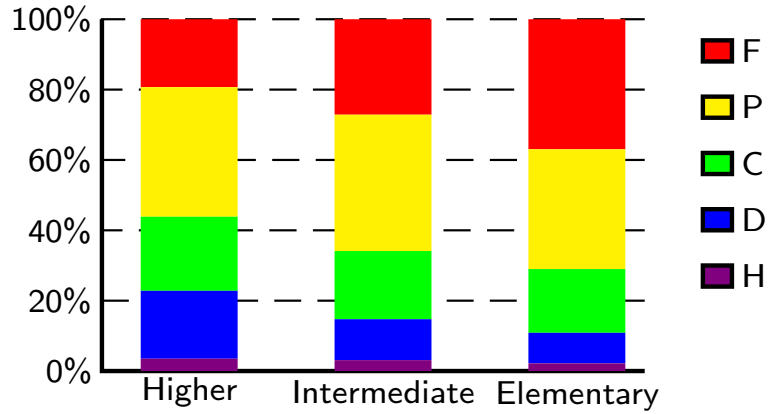
Nicholas & R 2015

Mathematics background is important

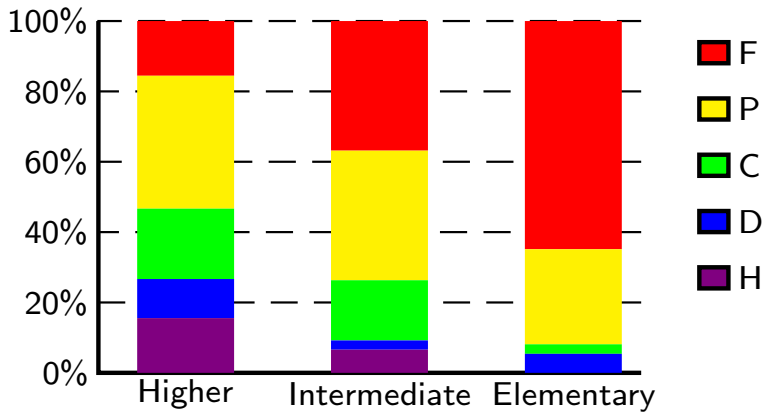
First-year subjects, all students who studied school maths the year before.



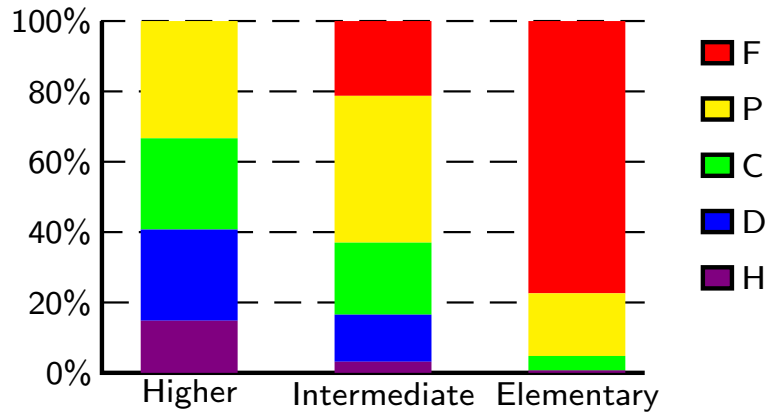
Discrete Mathematics (75)



Statistics (324)



Physics (170)



Basic Mathematics (310)

R & Coady 2009

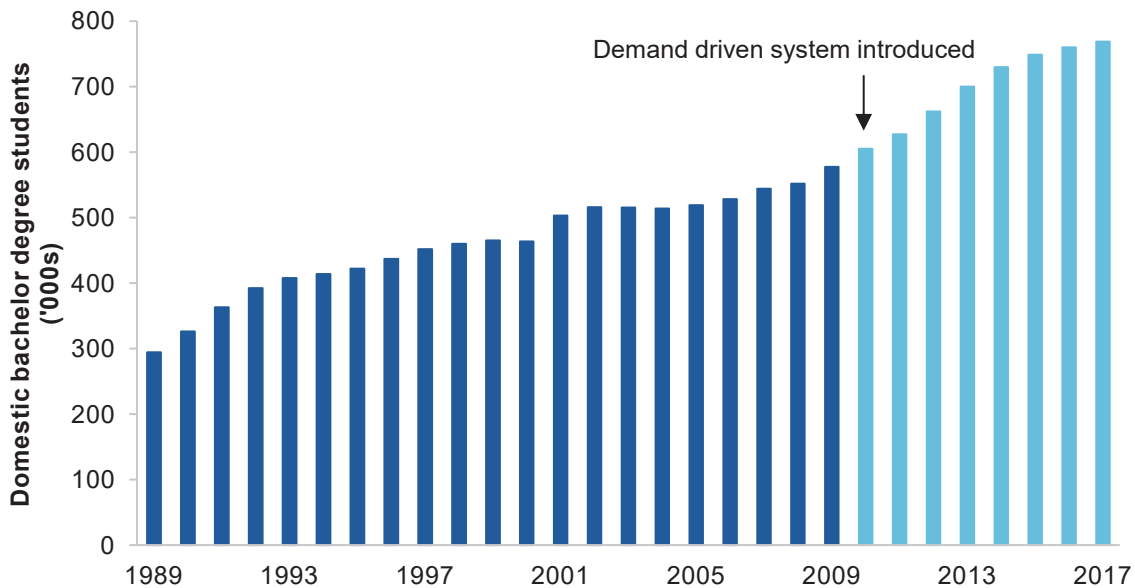
Growth in undergraduate numbers 1989–2017

Australian population (millions)

1989	2009	2017
16.8	21.7	24.6

Figure 2 **Enrolments grew during the demand driven system**

Domestic bachelor degree students



Productivity Commission. The Demand Driven University System: A Mixed Report (2019).

Less school maths + more undergraduates

2019, The Demand Driven University System

89% of the additional students attended non-Group of Eight universities. Additional students are also somewhat more likely to study education, IT or management and commerce, and are less likely to study engineering or science.

The additional students have, on average, weaker foundational skills. About 81% of additional students had numeracy below the average level of other students at age 15 years.

Together with the long-term decline in literacy and numeracy of Australian school students, this means that far more students are entering university ill-prepared than before the demand driven system. (p.35,36)

Productivity Commission

Beyond Australia

TIMSS Trends in International Mathematics and Science Study. Run every 4 years. An assessment of student achievement in mathematics and science in Grades 4 and 8.

We are below Kazakhstan.

Various media 2016

In 2015

- 4th grade: Australia was 28th of 49.
- 8th grade: Australia was 17th of 39.

In both 4th and 8th grades we are many places below Kazakhstan, and below many countries we'd like to be above.

Thomson et al. 2016, Figs 2.1, 3.1.

Mathematics over the past 20 years has flat-lined, while many other countries have improved.

ACER Media release 29 Nov 2016

`https://www.acer.org/au/timss/australian-results-timss-2015-dec-2016`

Beyond Australia

PISA Programme for International Student Assessment. Run every 3 years. An OECD survey that tests 15-year-old students.

PISA: 2012 to 2015 is the most recent part of a drop over a longer period of time. OECD 2016

Since 2003, Australia's results in maths have declined further than any country but Finland. Sydney Morning Herald 4 Dec 2019

2019: Australia is more than $3\frac{1}{2}$ years of lower than the highest performing economy (China), and around 3 years lower than the highest performing country (Singapore). (p.4) Thomson et al. 2019

Of 24 developed countries investigated, in only England, Scotland, Wales, Northern Ireland, Ireland and Australia is mathematics not compulsory after the age of 16. Lawson et al. 2019

The Australian Industry Group

2015: The Ai Group made a submission to the Decadal Plan for the Mathematical Sciences. It included "Further research is required to develop measures to raise the numeracy levels of the Australian workforce."

Ai Group submission: A Decadel Plan for the Mathematical Sciences

2018: The Ai Group is aware of the declining levels of school maths, and of the TIMSS and PISA results.

The Australian Industry Group, 2018

The ATAR (Australian Tertiary Admissions Rank)

2014 MANSW survey report: 1 084 responses from secondary teachers of mathematics in NSW (estimated 18% of NSW secondary mathematics teachers).

- The teachers believe that many capable students are choosing Elementary (General Mathematics) rather than Intermediate (Mathematics 2 Unit) to maximise their ATAR.
- One of the three most common reasons for choosing a mathematics course below their capability was a desire to optimise HSC and ATAR results. (p.3)
- 51% believe that a substantial number of mathematically able students in their school are selecting a senior mathematics course below their capability. Only 34% disagreed. (p.3)

Report on the MANSW Survey, 2014.

Prerequisites

1997 WSU: 3unit Mathematics or equivalent was required for the Bachelor of Computer Science.

2006: Lowering mathematics prerequisites is undermining enrolments in high school mathematics. (p.9) Rubinstein 2006 (AMSI)

2011: B Science (chemistry): of 17 universities, 7 had maths prerequisites, 5 assumed knowledge, 5 nothing. Belward et al. 2011.

2014: Prerequisites for degrees with significant mathematical requirements recommended. Report on the MANSW Survey, 2014.

2017: The reintroduction of clear maths prerequisites will emphasise the national and personal importance of mathematics and give us some relief from the insidious ATAR gaming that trips up many students. AMSI 2017

2017: One of 5 priorities: Restore university maths prerequisites from their historic low and turn around declining school mathematics enrolments.

AMSI 2017

Prerequisites

2018 Alan Finkel, Australia's Chief Scientist:

- "Now the federal Minister for Education has said very clearly: 'All Australian universities should reintroduce clear pre-requisites as part of their admissions processes, particularly requiring maths or science subjects, as well as English.'" CONASTA Keynote Address
- "If there's one thing that I hear more often than anything else, it's this: bring back mathematics prerequisites for courses where a knowledge of mathematics is required. I hear it from teachers, I hear it from parents, I hear it from employers and yes, I even hear it from your lecturers." Dinner Address, UA Ed Conference

2019: Univ. Sydney. The announcement of prerequisites in 2016 coincided with a reversal of trends 2016–2018. Enrolments remained high despite the introduction of prerequisites. Ramagge

Prerequisites: Univ. Sydney from 2019, the ANU from 2022.

Assumed knowledge

2015: There is a perception that assumed knowledge is misunderstood or ignored. Universities give different definitions.

King & Cattlin

Finkel: assumed knowledge from a Go8 website.

> *Assumed knowledge is not a requirement in order to apply, but helpful to have a background in the courses you'll be studying. If there is assumed knowledge that you don't have, you might like to consider doing some extra study, or even your own research to get up to speed.*

"You 'might like to consider' it?
Are we seriously suggesting that you can pick up calculus in your spare time without an expert teacher to guide you?
Assuming you feel like it?"

CONASTA Keynote Address

School mathematics teachers

Only teachers who are maths qualified or accredited should teach mathematics. Office of the Chief Scientist, 2012 (p.7)

The problem of not enough fully qualified maths teachers goes back to at least 1989. O'Connor & Thomas, 2019 (AMSI)

2007: The president of the Aust MS said that we were experiencing a debilitating shortage of adequately trained school mathematics teachers.

2017: At least 26% of Year 7 to 10 maths classes don't have a qualified maths teacher. The international average is around 12%. AMSI 2017

2019: Fewer than 25% of students have a qualified maths teacher for all of years 7–10. This will get worse as the estimated increase in school enrolments is 10,900 extra students each year.

O'Connor & Thomas, 2019 (AMSI)

School mathematics teachers

2017: For 20+ years there has been a reduced intake of mathematically qualified graduates into teacher training programs, fewer qualified secondary school teachers, . . . AMSI 2017

2018 Finkel "Yesterday, Minister Birmingham laid it out in the starkest terms. He said very clearly, and I will repeat it, because we ought to remember it:
It is unacceptable that secondary school students are taught science or maths subjects by people without specialist skills in science and maths.
The Minister has defined a new aspiration.

Every high school should have access to specialist teachers to teach science and maths subjects. And we should strive to achieve this within the next five to ten years."

CONASTA Keynote Address

Effects on university teaching

The main challenge for academics teaching first-year mathematics was the mathematical diversity of students. King & Cattlin 2015

Mathematics is a hierarchical subject. The ability to proceed to new work is very often dependent on a sufficient understanding of one or more pieces of work which have gone before.

There is a 'seven year difference' in achieving an understanding of place value which is sufficient to write down the number which is 1 more than 6399. Whereas an 'average' child can perform this task at age 11 but not at age 10, there are some 14 year olds who cannot do it and some 7 year olds who can. The Cockcroft Report 1982

There isn't room in many degrees for the number of subjects needed to lift the mathematics of students who are years behind.

Effects on university teaching

2012: English universities are side-lining mathematical content because students and staff lack the requisite confidence and ability.

Lawson et al. 2019

2013: At least 1 academic reduced quantitative content because of students' skills. (p.838) Rylands et al. 2013

2015:

- Pass grades were being lowered to maintain progression rates.
- High failure rates reflected poorly on their teaching ability or on the reputation of their department.
- Maths staff told that they're teaching poorly when trying hard to meet students' needs.
- Some academics had information about student enrolment numbers, but institutional reputation meant that they were unable to publish or document their concerns and findings. King & Cattlin 2015

Effects on university teaching

2017: Course realignments to cope with increasing numbers of less mathematically literate students. AMSI 2017

2019: Universities might inadvertently lower standards or discriminate against groups more likely to drop out. Productivity Commission 2019

A downward spiral? A danger of a self-perpetuating situation—students receiving undergraduate education in which mathematical content is side-lined later make up the bulk of academic staff and do not have the confidence to make the curriculum more mathematical. Lawson et al. 2019

2019: A report for the Minister for Education on performance based funding proposed 4 measures for the funding scheme, one of which retention. Wellings et al. 2019

The equity argument against prerequisites

Most universities have a variety of pathways for entry, including foundation programs and colleges. There are pathways into most universities for students without prerequisites.

Admitting students who can't pass is unethical.

Self-inflicted disadvantage is not an equity issue.

If there are too many poorly prepared students, standards drop.

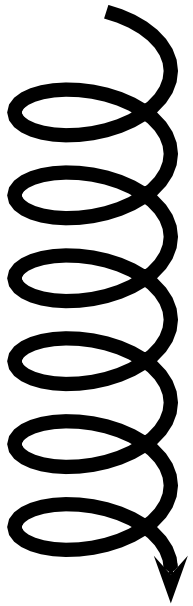
What do we want of the "50" student?

Should credit at university be given for school subjects? If so, how many?

What is best for Australia?

Consequences

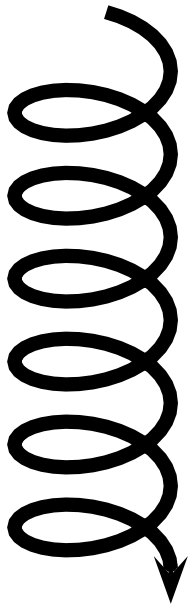
- A downward spiral in mathematical knowledge.
- Some mathematics staff are blamed for poor teaching.
- Less maths and/or lower levels of maths to keep retention rates acceptable.
- Some degrees far less mathematical than they used to be.
- Some feel they can't speak out about problems.



"Maths" degrees with little maths?

Consequences

- A downward spiral in mathematical knowledge.
- We are blamed for poor teaching.
- Remedial (elective) $\rightarrow$ core $\rightarrow$ dumbed down $\rightarrow$ replaced by easier subject $\rightarrow$ deleted.
- "Maths" degrees with not much maths.
- No knowledge of the situation beyond our group. Horror by those outside universities.
- Some degrees far less mathematical than they used to be.

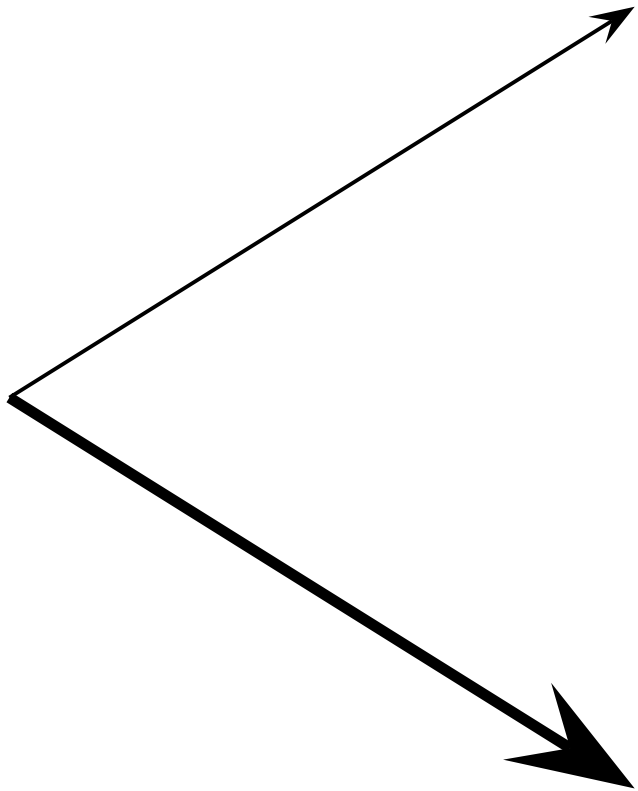


Service teaching

- Maths $\rightarrow$ lower retention/fewer students $\rightarrow$ fear of job loss $\rightarrow$ remove maths.
- Maths $\rightarrow$ many fail $\rightarrow$ "poor teaching" $\rightarrow$ others want to teach it ($).

Where to from here?

The mathematics problem



Why use a diagnostic test?

1. Inform teaching staff about the level of knowledge of students, perhaps enabling them to target their teaching to the level of (most) students.
2. Identify at-risk students, with the aim of providing assistance.
3. Inform students of any deficiencies in their knowledge so that they can then address these.
4. Enable students and/or staff to decide on the right level of subject for each student (in cases where there is a choice).
5. Require students to reach a determined level of skills in order to progress.
6. Predict performance.
7. Inform non-mathematicians and decision makers about the level of mathematical knowledge of students.

Diagnostic tests, an example

- It takes time to become proficient in addition and subtraction, fractions, . . . there is only one semester.
- Students will study what they expect to find in assessments.
  - If the test is on calculus, students study calculus, even if they can't add and subtract.
- An attempt at a solution: a diagnostic test in which students must score at least 80%. Otherwise they fail the subject.
- Six attempts across the semester.
  - With support workshops to build skills.

The test (no calculators)

- Find $-6 + 4 \times -5 - 3$.
- Find $\frac{2}{3} + \frac{3}{5}$.
- Round 27.48281 to 3 significant figures.
- Write $\frac{11}{20}$ as a percentage.
- Find $6.32 \div 100$.
- Find 4% of \$6000?
- Find $\frac{3}{8} \times 1\frac{1}{3}$.
- Arrange the following in ascending order (smallest to largest)

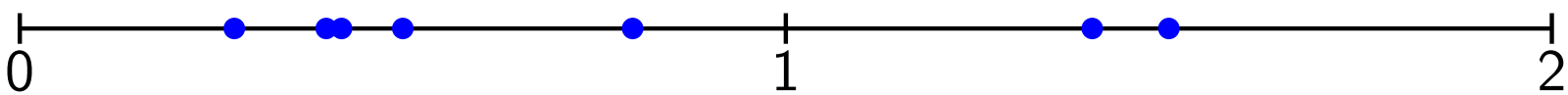
  0.702, 0.072, 0.72, 0.0702.
- Change 24.59 metres into centimetres.
- Write 0.000483 in scientific notation.
- Simplify $5x - 3w - 7x + 8w$.
- Simplify $3x \times 2xy$.
- Expand and simplify $3(a + 4) + 4a$.
- A photograph has length 24 cm and width 16 cm. It is to be enlarged so that its length becomes 36 cm. What will be the width?

Diagnostic tests, an example

- 56% did not reach the threshold.
- Almost a quarter of students
  - could not change a simple measurement from m into cm,
  - could not find 4% of \$6000,
  - could not divide an integer by 100;
- Over a quarter could not add 2 simple fractions;
- Over a third could not simplify $-6 + 4 \times -5 - 3$;

After a few attempts:
Some do not know where the decimal point goes in 17

Fifth test, attempted by eight students:
Mark on the number line where $\frac{2}{5}$ should be.



The outcome

- 18% had not reached the threshold by the end of semester. None of these would have passed, had they reached the threshold.
- A noticeable number of students spent time working on basic skills. Not all of these made it, but they did improve.
- "I feel as if I have learned something (finally)".

Was the test useful?

- It did improve the basic skills of some.
- Those who didn't reach the threshold failed anyway.
- I've told many about the results.

Students' expectations

Students' perspectives are different to those of staff.

- At the start of semester we see many poorly prepared students in our first-year subjects;
- We have high failure rates;
- What are students' expectations the start of semester?
- How do students expectations change over a semester?
- Do students' expectations align with reality?

We ran 2 surveys in four low level first-year subjects, one in the first week of semester and one at semester's end.

Number of responses: 1156.

We have the final grades.

Four first-year low level mathematics subjects

Students were surveyed on

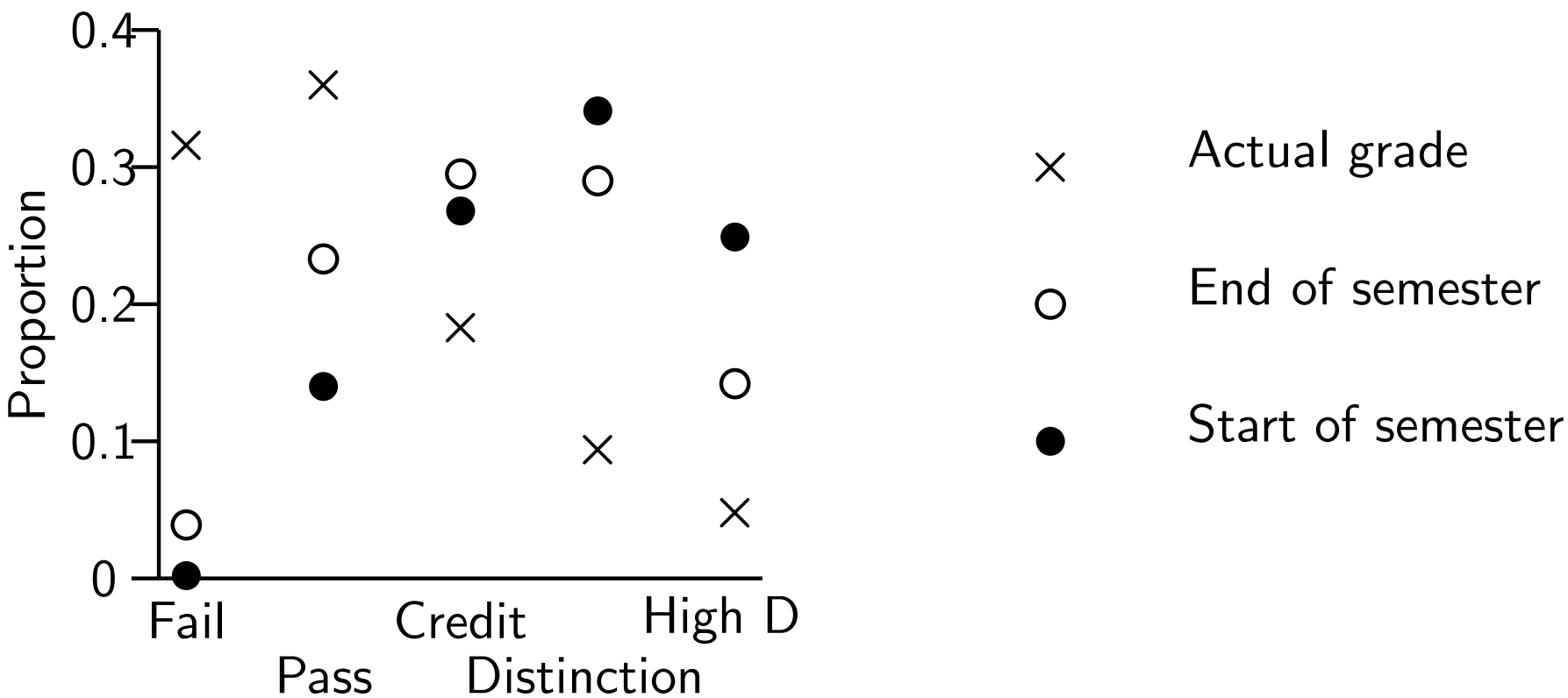
- Their mathematical background;
- Their attitude to mathematics;
- The grade they expect to get in their mathematics subject;

The first survey took place in the first week of semester, which for many was their first week at university.

We learnt that about half the students did not attend their first lecture.

We learnt that many incorrectly reported their high school maths.

Expected grades and actual grades



Students' expectations

- Poorly prepared students don't see themselves as poorly prepared. (72% had inadequate mathematics.)
- Most students expect a higher grade than what they achieved.
- Students' assessment of their skills and expected grade increased with increasing mathematics background.
- Do high expectations mean that students don't work very hard, believing that they will pass?
- Do students ignore support services because they believe they don't need them?

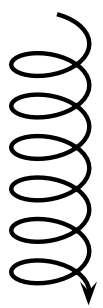
The mathematics problem

We have a problem. What can we do?

- Use existing evidence;
- Gather evidence;
- Increase understanding of the problem in our groups and beyond;
- Prerequisites;
- Tell everyone, management, industry, the world;
- Be honest and open about the problem.

What if

- maths graduates don't understand proof and struggle with algebra?
- statistics graduates think it reasonable to use 100 parameters to model a data set with 10 points?
- universities give credit for mid-early high school maths?



FYiMaths (First year in maths)

Our goal is to improve outcomes for students in undergraduate mathematics by sharing and developing teaching practices, conducting research and building connections between secondary schools and universities.

`https://fyimaths.weebly.com/`

Several events each year.

- Measuring the mathematics problem, June 2000. https://www.engc.org.uk/publications/.
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